THE UNIVERSITY OF ALBERTA

FORECASTING INTEREST RATES WITH EXPONENTIALLY SMOOTHED MOVING AVERAGES AND MULTIPLE WEIGHT SETS

by

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ABSTRACT

We demonstrate in this paper that the accuracy of exponentially smoothed forecasts is enhanced by employing multiple rather than single weight sets in the smoothing process. Specifically we show that during two time periods, each of about ten months duration, the variance as well as the average absolute deviation in daily forecasts of Federal Funds rates was reduced by permitting a separate weight vector (α,β,γ) for each week day. This is done in lieu of using a single weight vector (α,β,γ) in the smoothing process over the particular periods in question. It is this modification to the conventional exponential smoothing model with ratio seasonal factors and linear trend, that enabled an optimal weighting of past observations. We also show the accuracy obtained by an exponential smoothing model with the above modification, in forecasting daily interest rates during the second ten month period with the weights found optimal during the first ten month period. This is done to simulate an actual application of the forecasting model and to evaluate how well the forecasting model would have performed in practice.

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INTRODUCTION

The specific task of this paper is to develop a forecasting model for daily rates of U.S. Federal Funds. These are (essentially) checking account balances held by member banks of the U.S. Federal Reserve System with their district Federal Reserve Bank. We will be concerned with those member banks of the U.S. Federal Reserve System, who for purposes of reserve management engage in Federal Funds transactions. The forecasting model is designed specifically for this purpose. Such forecasts, however, could also be useful to other financial institutions who deal in Federal Funds.

The forecasting model to be developed is based upon the technique of exponential smoothing with ratio seasonal factors and linear trend. Several discussions of the technique of exponential smoothing are available in the literature.[1] Such a model has previously been presented by Christian T. L. Janssen.[2] However, Janssen employs the conventional exponential smoothing method with a single weight set. The contribution of this paper is the introduction of multiple weight

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sets into the conventional exponential smoothing model. The idea of multiple weight sets grew out of an observation that the forecast errors were generally larger on Wednesdays than on other days of the week.[3] It was then natural to attempt to increase the accuracy of the Wednesday forecasts by employing weights that were best relative to Wednesdays for the Wednesday forecasts. The idea was then extended to also include a separate weighting for each of the other week days. Using separate weights for different days should permit a better weighting of past observations in estimating the constant term, the seasonal factor and the trend element for each separate week day.

The optimal weight set is usually considered to be that which minimizes e.g. the variance in the forecast errors over some particular period in the past. The reason for this is of course that the loss function is not readily obtainable in most applications. The appropriateness of any one criterion in the absence of the loss function then becomes largely a matter of judgement. We will choose as the optimal weight set for each week day that which minimizes the average squared deviation as well as the average absolute deviation of the forecasts from their mean. We demonstrate that for the particular application presented in this paper, this leads to a reduction in the total forecast errors for both of the above criteria.

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[3] Wednesday is the last day of the reserve week and supply and demand conditions have generally been erratic. This has rendered Wednesdays a special problem for the forecasting process.

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METHODOLOGY

The forecasting model will be developed over two ten month periods. The first extends from 01/04/68 to 10/30/68 and the second period extends from 08/29/68 to 06/25/69.[4] It is sufficient for purposes of bank reserve management to have a forecast for the next day only. A decision model for reserve management using a forecast for the next day only is presented by Janssen.[5] Only one day forecasts will therefore be used. The development of the forecasting model involves choosing an optimal weighting scheme for each day of the reserve week and also a single weighting scheme for the particular period in question such that the average squared deviation (first criterion) and the average absolute deviation (second criterion) of the forecasts from their mean is minimized. We are interested in both criteria since it is difficult to determine the consequences of various errors in this particular application. Whenever the consequences of one large error are more serious than that of several small errors the average squared deviation would be more appropriate.

A search over a grid of possible values of the weights will be conducted with increments equal to 0.1 from 0 to 1 for α, β and γ. To establish the superiority of using separate weight vectors (α, β, γ) for different week days, we will compare the error variances obtained on each day. This comparison will be made by dividing up the error variance found when using a single weight set into its four component parts i.e., the error variance for Friday, Monday, Tuesday and Wednesday,

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[4] Ten weeks were used for the initialization of the forecasting process

[5] Janssen, op. cit.

and contrasting it with the error variance on each day using the weight set found optimal for that day.

We will then proceed to test the accuracy of the forecasting model by using the multiple weighting scheme found to be optimal in the first ten month period, to forecast the interest rates for the second ten month period. We know from previous analysis of the second ten month period what its optimal weighting scheme is and the resulting error variances, therefore we can compare these results with those generated using the optimal weighting scheme for the first ten month period. This is done in order to simulate an actual application of the forecasting model and to evaluate how well the forecasting model would have performed in practice.

THE FORECASTING MODEL

The assumptions that led to the use of exponential smoothing with ratio seasonals and linear trend were that the underlying process that generated the effective rate of Federal Funds is composed of a constant term, a seasonal factor and a trend element. The appropriateness of a multiplicative seasonal factor is based upon the belief that the amplitude of the seasonal factor is dependent upon the level of interest rates. Whereas had this not been the case and the amplitude of the seasonal factor were independent of the level of interest rates, an additive seasonal factor would have been more appropriate. For the trend no such dependence is believed to exist and an additive trend component is chosen.

Exponential smoothing is used to arrive at estimates of the constant term, the seasonal factor and the trend element. The smoothing function consists of a linear combination of all past observations and is such that the weight of each observation decreases geometrically with its age.[6] The greater the smoothing constant the less importance is given to past observations. A smoothing constant equal to zero, means that no revision takes place, and thus the initial estimate remains as the most current throughout the smoothing period. The present forecasting model uses three smoothing functions each to provide a different estimate for the constant term, the seasonal factor and the trend element. As the effective rate becomes available, a new estimate

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[6] This is most easily demonstrated for the case of single exponential smoothing, where the smoothing function has the following form,

$$b_{t+1} = \alpha \sum_{n=o}^{\infty} (1 - \alpha)^n a_{t-n}$$

is made each day. Thus it is the most recent estimate of the constant term, the seasonal factor and the trend element, which make up the next day forecast for the effective Federal Funds rate.

The mathematical relationships follow below.[7]

t : time index, t=1, (Thursday), 2, ..., 5, (Wednesday)

a_t : the effective rate on Federal Funds on day t .

$\bar{a}_t$: the seasonally adjusted estimate of the constant term on day t .

d_t : the estimate of the seasonal factor on day t .

e_t : the estimate of the trend element on day t .

b_{t+1} : the forecast made on day t of the Federal Funds rate for day $t+1$, $t=1, \ldots, 4$.
(No forecast is required for Thursday.)

α, β, γ : the smoothing weights for the revision of the estimates of the constant term, the seasonal factor and the trend element, respectively, $0 \le \alpha, \beta, \gamma \le 1$.

v_t : the forecast error on day t, $v_t = a_t - b_t$.

The estimates of the constant term, the seasonal factor, and the trend element are stated in equations (1,1), (1,2), (1,3) respectively. The normal reserve week has five business days, therefore the most

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[7] Janssen, op. cit. p. 17.

recent estimate of the cyclical factor available on day t is d_{t-5}, i.e., that obtained on the corresponding day during the preceding week.

$$(1\text{-}1) \quad \bar{a}_t = \alpha(a_t/d_{t-5}) + (1-\alpha)(\bar{a}_{t-1} + e_{t-1})$$

$$(1\text{-}2) \quad d_t = \beta(a_t/\bar{a}_t) + (1-\beta)d_{t-5}$$

$$(1\text{-}3) \quad e_t = \gamma(\bar{a}_t - \bar{a}_{t-1}) + (1-\gamma)e_{t-1}$$

The forecast for the next day is given by equation (1-4).

$$(1\text{-}4) \quad b_{t+1} = (\bar{a}_t + e_t)d_{t+1-5}$$

PERFORMANCE OF THE FORECASTING MODEL

The results obtained supported the expectations that the use of multiple weight sets as a modification to exponential smoothing, would improve the accuracy with which it was possible to forecast daily rates of U.S. Federal Funds, that is, reduce the error variance that was generated by a forecasting model with a single weight set. The results were favorable in each of the ten month periods. The error variances were consistently smaller when using a separate weight set for each different week day, than those obtained by using a single weight set. The average squared deviation was reduced by as much as 0.0415 for Wednesdays of the first ten month period; and as much as 0.2248 for Wednesdays of the second ten month period. The complete results appear in Tables 1 and 2. To indicate how well the model would have performed, we used the multiple weight sets found to be optimal for the first ten month period to forecast interest rates during the second ten month period. This resulted in an average squared deviation of .1986. If we had used the weight sets that were optimal for this period the average squared deviation would have been .1840. The difference, .0146, represents the error reduction that could have been generated had we been able to use the optimal weight sets for the second ten month period. Since this difference is small, we conclude that this procedure is reasonably accurate. The results when the comparisons are made using the average absolute deviation also appear in table 3. These results are similar to those discussed above.

We also used the single weight set found to be optimal for the first ten month period to forecast interest rates during the second ten

month period. The corresponding results are presented in table 4. In this case, the average squared deviation was .2630, but could have been again as low as .1840. The difference .0790 indicates that error reduction that could have been implemented had we been able to use the optimal weight sets for the second ten month period.

GENERAL COMMENTS ON THE EXPONENTIAL WEIGHTS

It is possible to make some statements about the optimal weights that we would expect to find associated with the Federal Fund rates series, that we have used.

The weighting parameter α is large in all cases such that the effect of older observations is quickly attenuated. This is generally the case, when the mean of the distribution changes quickly, and this would seem to be the case for our rate series. The weighting parameter β, that is the seasonal parameter, is small such that very little revision takes place, indicating little seasonal adjustments. However, the increase in the seasonal parameters for the second ten month period (see table 2) indicates greater adjustments are being made in the estimation process of the seasonal factor, corresponding to the higher levels of interest rates. Finally, the trend parameter makes only slight revisions in the smoothing process, except on Wednesdays. This indicates that there is an abrupt shift in the parameter, necessitating a quick reestimation of the trend element. The data (see tables 5 and 6) for Wednesday would seem to lend support to this, in that Wednesday's rate falls below the week's average almost seventy percent of the time. When $\gamma = 1$ the estimate of the trend is based solely upon the difference between the estimates of the mean on Tuesday and on Monday, (see equation, 1-3). This reflects the change in the trend from positive to negative in most instances starting on Tuesdays (see tables 5 and 6).

SUMMARY

The modification of using separate weight vectors (α,β,γ) for different week days in the smoothing process of the conventional exponential smoothing model with ratio seasonals and linear trend has reduced the forecast errors generated when trying to forecast daily Federal Funds rates.

The reduction in forecast errors was in evidence in both trial periods, and the accuracy of forecasting the second period rates while using first period weighting schemes was reasonably satisfactory.

It is therefore the contention of the author that there is enough supporting evidence to warrant stating the superiority of multiple weight sets over a single weight set in the process of exponential smoothing with ratio seasonals and linear trend for the particular application of this paper. Whether this would also hold in general cannot of course be inferred from the present study.

TABLE 1

The average squared deviation and the average absolute deviations for single and multiple weight sets respectively, for the first ten month period, 01/04/68 to 10/30/68

Average squared deviation:

Optimal weights for single weight set (α,β,γ) respectively are (0.8,0.4,0.1)

Day	Friday	Monday	Tuesday	Wednesday	Total
Optimal weights are:	(.8,.1,.1)	(.8,.1,.1)	(.8,.1,.1)	(.9,.1,.9)	
Single weight	.0512	.0272	.0631	.1853	.0820
Four weights	.0468	.0263	.0601	.1438	.0692
Difference	.0044	.0009	.0030	.0415	.0128

Average absolute deviation:

Optimal weights for single weight set (α,β,γ) respectively are (0.8,0.3,0.1)

Day	Friday	Monday	Tuesday	Wednesday	Total
Optimal weights are:	(.9,.1,.1)	(.9,.1,.1)	(.8,.1,.2)	(1.0,.1,.6)	
Single weight	.1471	.1155	.1880	.3145	.1913
Four weights	.1286	.1047	.1812	.2651	.1699
Difference	.0185	.0108	.0068	.0494	.0214

TABLE 2

The average squared deviation and the average absolute deviation for single and multiple weight sets respectively, for the second ten month period, 08/29/68 to 06/25/69

Average squared deviation:

Optimal weights for single weight set (α,β,γ) respectively are (0.9,0.6,0.1)

Day	Friday	Monday	Tuesday	Wednesday	Total
Optimal weights are:	(.9,.3,.1)	(.9,.2.1.)	(.9,.4,.1)	(.9,.5,1.0)	
Single weight	.0604	.0688	.0807	.7821	.2480
Four weights	.0421	.0612	.0752	.5573	.1839
Difference	.0183	.0076	.0055	.2248	.0641

Average absolute deviation:

Optimal weights for single weight set (α,β,γ) respectively are (0.9,0.4,0.1)

Day	Friday	Monday	Tuesday	Wednesday	Total
Optimal weights are:	(.9,.2,.1)	(.9,.3,.1)	(.7,.7,.1)	(.9,.5,1.0)	
Single weight	.1775	.1765	.1958	.6619	.3029
Four weights	.1578	.1734	.1805	.5392	.2628
Difference	.0197	.0031	.0153	.1227	.0401

TABLE 3

The average squared deviation and average absolute deviation generated throughout the second ten month period from 08/29/68 to 06/25/69, indicating how well the model would have performed in practice.

Average squared deviation:

Day	Friday	Monday	Tuesday	Wednesday	Total
Optimal weights for first ten month period are:	(.8,.1,.1)	(.8,.1,.1)	(.8,.1,.1)	(.9,.1,.9)	
First period weights	.0522	.0618	.0760	.6043	.1986
Optimal weights for second ten month period are:	(.9,.3,.1)	(.9,.2,.1)	(.9,.4,.1)	(.9,.5,.1.0)	
Second period weights	.0421	.0612	.0752	.5573	.1840
	.0101	.0006	.0008	.0470	.0146

Average absolute deviation:

Day	Friday	Monday	Tuesday	Wednesday	Total
Optimal weights for first ten month period are:	(.9,.1,.1)	(.9,.1,.1)	(.8,.1,.2)	(1.,.1,.6)	
First period weights	.1665	.1938	.1988	.5843	.2859
Optimal weights for second ten month period are:	(.9,.2,.1)	(.9,.3,.1)	(.7,.7,.1)	(.9,.5,1.0)	
Second period weights	.1578	.1734	.1805	.5392	.2627
	.0107	.0204	.0183	.0451	.0332

TABLE 4

The average squared deviation and average absolute deviation generated throughout the second ten month period from 08/29/68 to 06/25/69, indicating how well the model would have performed in practice.

Average squared deviation:

Optimal weight set, (α,β,γ) respectively are, (0.8,0.4,0.1)

Day	Friday	Monday	Tuesday	Wednesday	Total
Optimal weight set for second ten month period:	(.9,.3,.1)	(.9,.2,.1)	(.9,.4,.1)	(.9,.5,1.0)	
First period weights (one weight set)	.0686	.0775	.0850	.8207	.2630
Second period weights (multiple weight set)	.0421	.0612	.0752	.5573	.1840
	.0265	.0163	.0098	.2634	.0790

Average absolute deviation:

Optimal weight set, (α,β,γ) respectively are, (0.8,0.3,0.1)

Day	Friday	Monday	Tuesday	Wednesday	Total
Optimal weight set for second ten month period:	(.9,.2,.1)	(.9,.3,.1)	(.7,.7,.1)	(.9,.5,1.0)	
First period weights	.1945	.1962	.2065	.6857	.3207
Second period weights	.1578	.1734	.1805	.5392	.2627
	.0367	.0228	.0260	.1465	.0580

TABLE 5

Effective Rates on Federal Funds for the first ten month period 01.04.68-10.30.68. (Source: The Federal Reserve Bank of New York.)

THU	FRI	MON	TUE	WED
4.625	4.750	4.625	4.625	4.250
4.625	4.500	4.500	4.750	4.875
4.750	4.750	4.625	4.625	4.625
4.625	4.500	4.500	4.500	4.750
4.750	4.750	4.750	4.750	4.625
4.750	4.750	4.750	4.750	4.625
4.750	4.750	4.625	4.625	4.625
4.625	4.875	4.750	4.750	4.500
4.750	4.750	4.875	4.875	5.000
5.000	5.000	5.000	4.500	3.000
5.500	5.125	5.250	5.500	4.250
5.500	5.500	5.375	5.000	5.375
5.750	5.250	5.500	5.750	5.875
5.875	5.750	5.500	5.500	5.500
5.750	5.750	5.750	5.750	5.500
5.750	5.750	5.875	5.500	5.000
5.875	6.125	6.125	6.250	6.375
6.250	6.125	6.125	6.000	5.500
6.250	6.250	6.375	6.500	6.500
6.375	6.375	6.375	5.875	5.000
6.250	6.125	6.000	5.750	5.750
5.750	6.125	6.125	5.750	5.500
6.250	6.125	6.250	6.375	6.000
6.250	6.125	6.000	6.125	6.500
6.375	6.375	6.500	5.750	5.250
6.500	5.500	6.125	5.500	5.500
5.500	6.000	6.000	6.125	6.000
6.000	6.000	6.250	6.375	6.250
6.125	6.125	6.125	6.000	6.125
6.125	6.125	6.000	5.875	6.000
6.000	6.125	6.125	6.250	6.000
6.125	6.125	6.125	6.250	6.000
6.125	6.125	6.000	6.000	5.875
6.000	5.875	5.875	6.000	6.000
5.875	5.750	5.750	5.750	6.000
6.000	6.000	6.000	5.875	5.000
5.750	5.750	5.750	5.625	5.385
5.500	5.625	5.750	5.875	5.875
6.000	6.000	5.500	6.000	6.000
6.000	5.875	6.000	5.875	6.000
6.000	6.125	6.125	6.000	5.250
5.875	5.750	5.750	5.875	6.125
6.000	6.000	5.875	5.875	5.625

TABLE 6

Effective Rates on Federal Funds for the second ten month period 08.29.68-06.25.69. (Source: The Federal Reserve Bank of New York.)

THU	FRI	MON	TUE	WED
5.875	5.750	5.750	5.750	6.000
6.000	6.000	6.000	5.875	5.000
5.750	5.750	5.750	5.625	5.385
5.500	5.625	5.750	5.875	5.875
6.000	6.000	5.500	6.000	6.000
6.000	5.875	6.000	5.875	6.000
6.000	6.125	6.125	6.000	5.250
5.875	5.750	5.750	5.875	6.125
6.000	6.000	5.875	5.875	5.625
6.000	6.000	6.125	6.125	6.125
6.125	6.125	6.125	6.125	5.250
6.000	6.000	6.000	5.625	2.500
5.500	5.750	5.825	5.750	5.750
5.750	6.000	6.125	5.875	4.500
5.875	5.875	5.875	5.875	5.625
5.875	6.125	6.000	5.875	5.875
6.125	6.250	6.375	6.375	6.375
6.500	6.875	6.500	6.500	4.000
6.500	6.750	6.625	6.625	5.000
6.500	6.500	6.625	6.375	5.500
6.625	6.625	6.250	6.250	6.250
6.500	6.375	6.375	6.125	5.750
6.250	6.375	6.375	6.250	6.250
6.750	6.750	6.750	6.750	6.750
6.750	7.250	7.000	6.750	5.000
6.750	6.625	6.500	6.500	5.750
6.625	7.000	6.750	6.625	6.250
6.875	6.875	6.625	6.625	6.500
6.875	6.875	6.875	6.750	6.625
7.000	7.000	6.750	6.750	6.625
6.875	6.875	6.250	6.625	6.250
6.875	7.000	7.125	7.375	6.875
7.375	7.625	7.500	7.875	7.750
7.750	7.875	7.500	6.750	6.750
7.750	7.750	7.750	7.750	8.000
8.250	8.500	8.250	7.875	7.750
8.375	9.000	8.000	7.750	7.000
8.500	9.250	8.875	8.750	8.500
9.000	9.250	8.750	8.750	8.250
9.375	9.375	9.375	9.000	8.500
9.625	9.875	9.500	9.125	6.000
9.500	9.250	8.500	7.750	6.250
9.000	8.750	9.125	8.000	6.000

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APPENDIX

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C     DEFINITION OF TERMS
C     WE=CONSTANT WEIGHT
C     WF=SEASONALITY WEIGHT
C     WA=TREND WEIGHT
C      Y(I,J)=THE EFFECTIVE RATE FOR WEEK I,AND DAY J.
C     FORC(I,J)=THE FORECAST RATE
C     FORC(I,J-1)=THE MOST RECENT FORECASTED RATE
C     Y(I,J-1)=THE ACTUAL RATE CORRESPONDING TO FORC(I,J-1)
C     A(I,J)=THE ESTIMATE OF THE TREND
C     F(I,J)=THE ESTIMATE OF THE SEASONALITY
C     SBAR(I,J)=THE ESTIMATE OF THE SEASONALLY ADJUSTED RATE.
C      THURSDAY=1,FRIDAY=2,MONDAY=3,TUESDAY=4,WEDNESDAY=5
C DATA FROM 08/29/68  TO 06/25/69 FEDERAL FUNDS EFFECTIVE RATES
      REAL B,BB,BBB,WE,WA,WF
      DIMENSION Y(43,5),FORC(43,5),A(43,5),F(43,5),SBAR(43,5),
     *YBAR(43),FL(43,5),S(43,5),F(43,5),EC(43,5),DS(43,5),T(43)
     *,AVEDAY(5),ECDAY(43,5),DSDAY(43,5),ADSDAY(5),AADDAY(5)
      K=43
      WRITE(6,2)
    2 FORMAT('-',39X,'THU',4X,'FRI',4X,'MON',4X,'TUE',4X,'WED',13X,'THU'
     1,4X,'FRI',4X,'MON',4X,'TUE',4X,'WED')
C INPUT DATA
      READ(5,3)((Y(I,J),J=1,5),I=1,K)
    3 FORMAT(8X,F4.3)
      WRITE(6,4)((Y(I,J,J=1,5),I=1,K)
    4 FORMAT('0','ACTUAL RATES',24X,5F7.3,10X,5F7.3)
C     INITIALIZATION PROCEDURE.
      DO 5 I=1,K
      YBAR(I)=0.0
      DO 6 J=1,5
      YBAR(I)=YBAR(I)+Y(I,J)/5
 6    CONTINUE
 5    CONTINUE
      WRITE(6,7)(YBAR(I),I=1,6)
 7    FORMAT(6X,'YBAR=',F5.3)
      DO 8 I=1,42
      T(I)=(YBAR(I+1)-YBAR(I))/5
 8    CONTINUE
      DO 9 I=1,K
      DO 10 J=1,5
      C=J
      S(I,J)=Y(I,J)
      IF(I.EQ.43)GO TO 10
      SBAR(I,J)=YBAR(I)
      A(I,J)=T(I)
      FL(I,J)=Y(I,J)/(YBAR(I)-(2.5-C)*T(I))
      F(I,J)=FL(I,J)
 10   CONTINUE
 9    CONTINUE
```

```
      WRITE(6,11)((FL(I,J),J=1,5),I=1,5)
 11   FORMAT(6X,'FL(I,J)=',5F7.3)
      WRITE(6,20)
 20   FORMAT(10X,'WE',5X,'WF',5X,'WA',5X,'AE',10X,'ADS',10X,'AABD',
     *10X,'EF',10X,'EM',10X,'ET',10X,'EW')
C     REVISION OF EXPECTED DESEASONALIZED RATE,SEASONAL FACTOR AND
C      TREND.
      DO 16 IJ=1,10
      DO 16 II=1,10
      DO 16 III=1,10
      B=IJ
      BB=II
      BBB=III
      B=B/10
      BB=BB/10
      BBB=BBB/10
      WE=B
      WF=BB
      WA=BBB
      SE=0.0
      DO 12 I=2,K
      DO 12 J=1,5
      IF(J.NE.1)GO TO 30
C     REVISION FOR FORECASTS FOR FRIDAY.
      SBAR(I,1)=(WE*S(I,1))/F(I-1,1)+(1-WE)*(SBAR(I-1,5)+A(I-1,5))
      F(I,1)=(WF*S(I,1))/SBAR(I,1)+(1-WF)*F(I-1,1)
      A(I,1)=WA*(SBAR(I,1)-SBAR(I-1,5))+(1-WA)*A(I-1,5)
      GO TO 31
 30   CONTINUE
C     REVISION FOR FORECASTS FOR MONDAY, TUESDAY,WEDNESDAY.
      SBAR(I,J)=(WE*S(I,J))/F(I-1,J)+(1-WE)*(SBAR(I,J-1)+A(I,J-1))
      F(I,J)=(WF*S(I,J))/SBAR(I,J)+(1-WF)*F(I-1,J)
      A(I,J)=WA*(SBAR(I,J)-SBAR(I,J-1))+(1-WA*A(I,J-1)
 31   CONTINUE
      DO 14 N=1,1
      C=N
C     FORECASTS FOR NEXT DAY.
      IF(J.EQ.5) GO TO 15
      FORC(I,J+N)=(SBAR(I,J)+C*A(I,J))*F(I-1,J+N)
C     CALCULATIONS FOR THE FORECAST ERRORS.
      E(I,J+N)=S(I,J+N)-FORC(I,J+N)
      SE=SE+E(I,J+N)
      GO TO 14
 15   CONTINUE
C     FORECASTS FOR THURSDAY.
      FORC(I,1)=(SBAR(I,5)+C*A(I,5))*F(I-1,1)
 14   CONTINUE
 12   CONTINUE
C     CALCULATIONS FOR THE AVERAGE ERROR.
      AE=SE/(K*4-4)
      SDS=0.0
      SABD=0.0
```

```
      DO 17 N=1,1
      DO 17 I=2,K
      DO 17 J=1,5
      IF(J.EQ.5)GO TO 17
C     CALCULATIONS FOR THE FORECAST DEVIATIONS.
      EC(I,J+N)=E(I,J+N)-AE
      DS(I,J+N)=(E(I,J+N)-AE)**2
      SDS=SDS+DS(I,J+N)
      SABD=SABD+ABS(EC(I,J+N))
 17   CONTINUE
C     CALCULATIONS FOR THE AVERAGE DEVIATION SQUARED AND THE AVERAGE
C     ABSOLUTE DEVIATION.
      ADS=SDS/(K*4-4)
      AABD=SABD/(K*4-4)
      WRITE(6,19)WE,WF,WA,AE,ADS,AABD
 19   FORMAT(9X,F3.1,3X,F3.1,3X,F3.1,3X,F7.4,5X,F7.4,7X,F7.4)
C     CALCULATIONS FOR AVERAGE DAILY ERRORS.
      DO 25 N=1,1
      DO 24 J=1,5
      SDAILY=0.0
      DO 23 I=2,K
      IF(J.EQ.5)GO TO 24
      SDAILY=SDAILY+E(I,J+N)
 23   CONTINUE
      AVEDAY(J+N)=SDAILY/(K-1)
 24   CONTINUE
 25   CONTINUE
      DO 26 N=1,1
      DO 27 J=1,5
      SUMDS=0.0
      SUMABD=0.0
      DO 28 I=2,K
      IF(J.EQ.5)GO TO 27
      ECDAY(I,J+N)=E(I,J+N)-AVEDAY(J+N)
      DSDAY(I,J+N)=(E(I,J+N)-AVEDAY(J+N))**2
      SUMDS=SUMDS+DSDAY(I,J+N)
      SUMABD=SUMABD+ABS(ECDAY(I,J+N))
 28   CONTINUE
      ADSDAY(J+N)=SUMDS/(K-1)
      AADDAY(J+N)=SUMABD/(K-1)
 27   CONTINUE
 26   CONTINUE
      WRITE(6,32)(AVEDAY(J),J=2,5)
 32   FORMAT(66X,'AE=',4F15.4)
      WRITE(6,33)(ADSDAY(J),J=2,5)
 33   FORMAT(66X,'ADS=',4F15.4)
      WRITE(6,34)(AADDAY(J),J=2,5)
 34   FORMAT(66X,'AABD=',4F15.4)
 16   CONTINUE
      STOP
      END
```